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written

$$[a^2 - k^2 \cos 2\beta]x^2 - 2k^2 \sin 2\beta \cdot xy + [a^2 + k^2 \cos 2\beta]y^2 = \frac{a^4 - k^4}{2}.$$

This is the equation of a system of conics, where the distance from the center to the focus is k, and where β is the angle between the major axis and the x-axis.

Also solved by ELIJAH SWIFT, who obtained the result $\phi = \int \frac{dr}{r\sqrt{cr^2(r^2 - a) - 1}}$, and by C. N. SCHMALL and who obtained $xy = c$.

The above solution by Professor HARDING is correct.

CALCULUS.

345. Proposed by C. N. SCHMALL, New York City.

Of all quadrilaterals which can be formed from four given sides, that which is inscriptible in a circle has the greatest area. [GOURSAT-HEDRICK, page 133, example 5.]

Proposer's Remark. Also show that when only three sides are known the length of the fourth side of the maximum quadrilateral is the root of a cubic equation.

SOLUTION BY THE PROPOSER.

Let the area $= u = \Delta ABC + \Delta ADC$. That is,

$$u = \tfrac{1}{2}(ab \sin \phi + cd \sin \psi). \tag{1}$$

Also

$$\overline{AC}^2 = a^2 + b^2 - 2ab \cos \phi$$

$$= c^2 + d^2 - 2cd \cos \psi.$$

Hence

$$a^2 + b^2 - 2ab \cos \phi = c^2 + d^2 - 2cd \cos \psi. \tag{2}$$

Now for a maximum we have from (1)

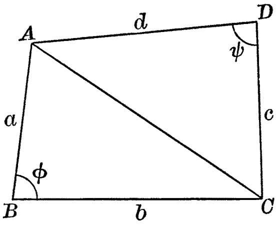
$$\frac{\partial u}{\partial \phi} = \frac{1}{2}\left(ab \cos \phi + cd \cos \psi \frac{\partial \psi}{\partial \phi}\right) = 0, \tag{3}$$

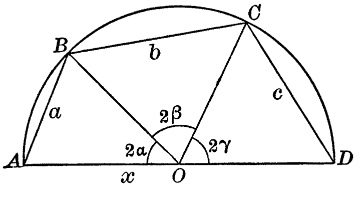
and from (2)

$$0 = 2ab \sin \phi - 2cd \sin \psi \frac{\partial \psi}{\partial \phi}. \tag{4}$$

Equating the values of $\partial\psi/\partial\phi$ obtained from (3) and (4), and reducing, we get

$$\frac{\cos \phi}{\sin \phi} = -\frac{\cos \psi}{\sin \psi}.$$





Hence $\tan \phi = -\tan \psi$, and $\phi + \psi = \pi$; that is, the opposite angles of the maximum quadrilateral are supplementary and hence the quadrilateral is cyclic.

Again, it is shown by elementary geometry, that when three sides of a maximum quadrilateral are given, the fourth side is the diameter of the circumscribing circle of the figure. Hence, to find this diameter AD, let $AD = x$, $AB = a$, $BC = b$, $CD = c$; and let the angles subtended by a, b, c, at the center be 2α, 2β, 2γ.

Then

$$\alpha + \beta + \gamma = \frac{\pi}{2} \tag{1}$$

and

$$\sin (\alpha + \beta + \gamma) = 1,$$

that is,

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + 2 \sin \alpha \sin \beta \sin \gamma = 1. \tag{2}$$

But $\sin \alpha = a/x$; $\sin \beta = b/x$; $\sin \gamma = c/x$. Substituting these values in (2), we have

$$x^3 - x(a^2 + b^2 + c^2) - 2abc = 0. \tag{3}$$

This cubic has two negative roots and one positive root, as may be seen by putting $x = 0, -a, -\infty$. The positive root is the value of x required. We can then describe a semicircle having this root as a diameter and place in it the chords a, b, c, in any order; for the equation (3) involves a, b, c, symmetrically.

Note.—The cubic (3) can also be derived as follows:

Draw the diagonals AC and BD. Then

$$AC = \sqrt{x^2 - c^2}, \qquad BD = \sqrt{x^2 - a^2}.$$

Hence, by *Ptolemy's Theorem* we have

$$\sqrt{x^2 - c^2} \cdot \sqrt{x^2 - a^2} = ac + bx,$$

whence

$$x^3 - (a^2 + b^2 + c^2)x - 2abc = 0,$$

as before.

Also solved by ELIJAH SWIFT, ELMER SCHUYLER, and A. M. HARDING.

MECHANICS.

273. Proposed by F. P. MATZ, Reading, Pa.

A person is placed on a perfectly smooth surface. How may he get off.

SOLUTION BY S. W. REAVES, University of Oklahoma.

He should throw some object, for example his hat, in the opposite direction to that in which he wishes to go. The reaction, or "recoil," will cause him to slide to the edge of the smooth surface.

284. Proposed by C. N. SCHMALL, New York, N. Y.

A cylindrical vessel standing upright on a horizontal plane is kept constantly full of water by an automatic device. Determine at what height in its side a small orifice should be made,